Film Thickness, Entrainment, and Pressure Drop in Upward Annular and Dispersed Flow

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Correlations for prediction of film thickness, entrainment, and pressure drop are required for modeling of annular and dispersed flow in tubular heat transfer and reactor systems. Existing correlations are rather poor predictors of film thickness and entrainment. The relatively large quantity of experimental data for upward flow at isothermal conditions provides the basis to develop these correlations. This note presents a design method that has been developed and used in a computer program to predict these variables from the input variables of the mass flow rates of the gas and liquid phases, tube diameter, and fluid physical properties.

FILM THICKNESS

Calvert and Williams (1958) and Anderson and Mantzouranis (1960) developed models for upward film flow conditions. Collier and Hewitt (1961) used the data of Bennett and Thornton (1961) to show that large deviations existed from the earlier models at low liquid flow rates, probably because of failure to allow for variation of shear stress in the film. Kosky (1971) used the horizontal flow data of McManus (1956), the vertical upflow data of McLeod (1951), Belkin (1953), and Charvonia (1959), and the vertical downflow data of Chien (1961) as well as the data used by Collier and Hewitt to develop a film thickness correlation. This shows the dimensionless film thickness $y_L{}^+$ as a function of the liquid film Reynolds number. Kosky used the wall shear stress for the shear velocity term in the dimensionless film thickness. The wall and interface shear stress for vertical upward annular gas-liquid flow are represented by the equations

$$\tau_w = \left[\left(-\frac{dp}{dL} \right) - g\,\rho_L \left(1 - \frac{a^2}{R_2} \right) - g\,\rho_G \frac{a^2}{R^2} \right] \frac{R}{2} \tag{1}$$

$$\tau_i = \left[\left(-\frac{dp}{dL} \right) - g\,\rho_G \right] \frac{a}{2} \tag{2}$$

At high gas velocities, the liquid density term of Equation (1) is small in comparison to the pressure loss with the result that the two shear stresses are nearly equal. At the lower limit of the annular flow regime, the wall shear stress is much less than the shear stress at the interface. Figure 1 shows low velocity annular flow data used by Collier and Hewitt with y^+ based upon the shear velocity corresponding to the wall shear stress and the average of the wall and interface shear stresses. High velocity data are also shown in which the two shear stresses are nearly equal. The mean shear velocity is observed to give better agreement with the high velocity data and with the film thickness corresponding to the velocity profile for turbulent flow.

Figure 2 shows the vertical upflow data used by Kosky with the exception of the Chien data for downflow and with the additional data of Alia et al. (1966). Horizontal flow data appear to be similar to upflow data, but downflow data show a somewhat different film thickness response and so were excluded from this analysis for an upflow model. The Bennett and Thornton data have been recalculated using the average shear stress in accordance with the results of Figure 1. The dimensionless film thickness is shown as a function of the integrated velocity profile with good agreement in the wall region. An empirical fit of the data provides

$$2 < Re_L < 100, \qquad y_L{}^+ = 0.66 \quad (Re_L)^{0.53} \tag{3a}$$

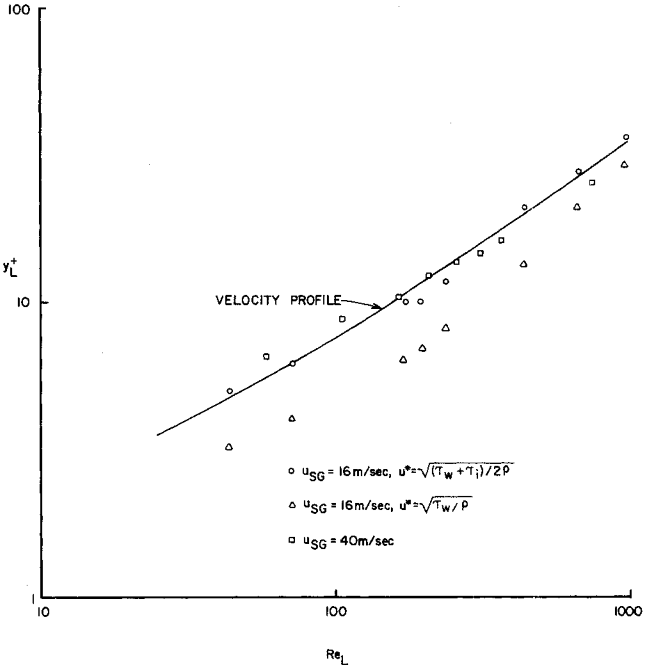

Fig. 1. Film thickness.

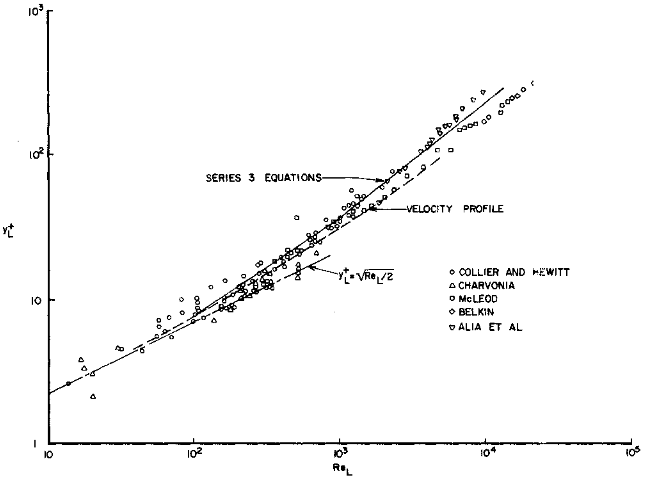

Fig. 2. Film thickness.

$$100 < Re_L < 1000, \quad y_L^+ = 0.347\ (Re_L)^{2/3} \qquad (3b)$$

$$1000 < Re_L \qquad y_L^+ = 0.13\ \ (Re_L)^{0.81} \qquad (3c)$$

ENTRAINMENT

Wicks and Dukler (1960) proposed an entrainment correlation for annular flow which was compared with upward flow data by Collier and Hewitt. They report good agreement with the correlation at high entrainment but systematic deviation at low entrainment rates. Entrainment might be expected to be a function of the film thickness. Figure 3 shows an entrainment correlation with the dimensionless film thickness corresponding to the gas phase shear velocity and kinematic viscosity. It is interesting to observe that entrainment appears to begin at a minimum y_G^+ of about 35 which is the transition between partially turbulent and fully turbulent conditions in single phase turbulent flow. The quantity of entrainment is shown to be represented by the volumetric flow ratio, $\alpha = W_E\rho_G/W_G\rho_L$. This term was used by Gill, Hewitt, and Lacey (1964) in analysis of entrainment data. Empirical correlating equations are

$$y_G^+ < 36, \quad \alpha = 0 \qquad (4a)$$

$$36 < y_G^+ < 42, \quad \alpha = -\ 0.000442 + 0.000013 y_G^+ \qquad (4b)$$

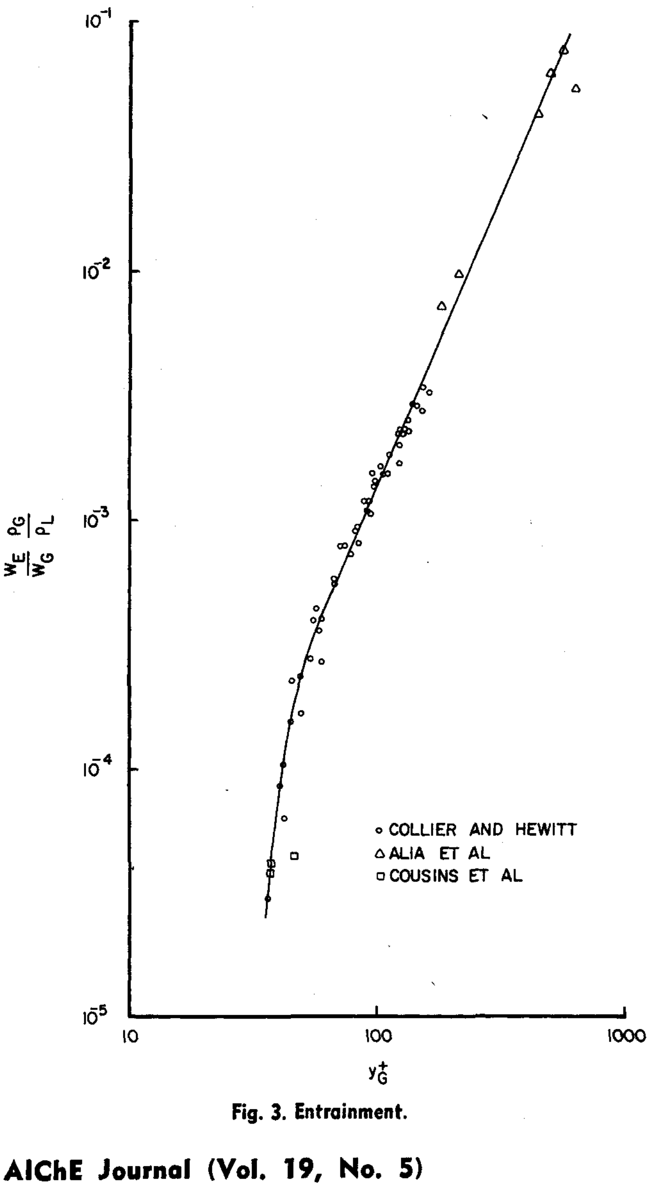

Fig. 3. Entrainment.

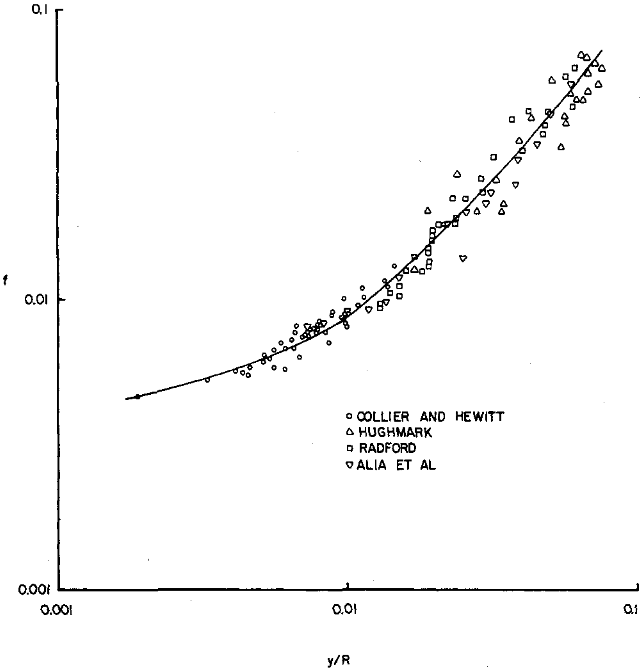

Fig. 4. Friction factor.

$$42 < y_G^+ < 60, \quad \alpha = -\ 0.000625 + 0.0000172 y_G^+ \qquad (4c)$$

$$60 < y_G^+ \quad , \quad \alpha = 5 \times 10^{-8} (y_G^+)^{2.2} \qquad (4d)$$

The experimental data used by Collier and Hewitt and the data of Alia et al. and Cousins, Denton, and Hewitt (1965) are used for this correlation.

PRESSURE DROP

Correlation of pressure drop as roughness height with the film thickness is presented by Gill, Hewitt, and Lacey. Hoogendoorn and Welling (1965) presented horizontal flow data with the conclusion that the effect of waves on the friction factor will depend on film thickness relative to the pipe diameter. Wallis (1969) has also suggested this friction factor model. Figure 4 show a correlation for upward flow data based upon the proposal of Hoogendoorn and Welling and Wallis. Friction factor is defined by

$$f = \frac{dP/dL\ D_c}{2 u_G^2 \rho_G} \qquad (5)$$

which is the friction factor for the gas core velocity with the core diameter. The data from Collier and Hewitt are for film thickness and pressure drop at the same conditions. The film thickness data of Alia et al. correspond to the pressure drop data of Cravarolo et al. (1964). The Radford (1949) and Hughmark (1959) data are for liquid holdup and pressure drop at atmospheric pressure discharge conditions. Liquid holdups are assumed to correspond to the liquid film thickness as the entrainment contribution to holdup is negligible at these conditions. The Hughmark data represent Varsol, aqueous sodium carbonate, and a 5 cp oil in addition to water as the liquid phase.

The Wallis model is similar to Equation (5) except that the pipe diameter is used in place of the core diameter. The friction equation approximates to

$$f = 0.005 \left(1 + 149 \frac{y}{R}\right)$$

TABLE 1. COMPARISON OF CALCULATED AND EXPERIMENTAL

		dP/dL		R_L	
Reference	System	No. of data points	% Avg. abs. dev.	No. of data points	% Avg. abs. dev.
Alía et al. (1965)	argon-water	—		34	13.0
Cravarolo et al.	argon-water	15	6.2	—	
Collier and Hewitt	air-water	100	5.1	99^3	7.8
Gill, Hewett, and Lacey	air-water	45	6.8	46^3	13.8
Radford	air-water	46	16.9	41	8.0
Semenov (1961)	steam^1-water	—		38	16.9
Hughmark	air-liquid^2	45	18.1	45	17.3

[1] 284, 570, and 1700 lb/sq. in. abs.

[2] Varsol, sodium carbonate solution, 5 cp oil, and water

[3] v/R data

SOLUTION PROCEDURE

Dimensionless film thickness and entrainment can be calculated by trial and error solution for distribution of liquid in the film and entrainment with the series 3 and 4 equations and the relationship from the dimensionless film thicknesses.

$$y_G{}^+ = (\rho_G/\rho_L)^{1/2}\,(\mu_L/\mu_G)\,y_L{}^+$$

Equations for dimensionless liquid film thickness and the shear velocity with Equation (5) result in the relationship

$$\frac{y}{R}\;\frac{(f/2)^{1/2}}{(1-2y/R)(1-y/R)^{1/2}} = \frac{y_L{}^+\mu_L}{R\,u_{SG}(\rho_G\rho_L)^{1/2}} = \phi \qquad (6)$$

Figure 4 provides the relationship for y/R and f, so y/R can be obtained as a function of the right side of Equation (6). The empirical equations are

$$\phi < 0.00069, \quad y/R = 3.85\,\phi^{0.815} \qquad (7a)$$

$$0.00069 < \phi < 0.0057, \quad y/R = 1.18\,\phi^{0.655} \qquad (7b)$$

$$0.0057 < \phi, \quad y/R = 0.78\,\phi^{0.577} \qquad (7c)$$

Thus y/R can be calculated from the series 7 equations and f from Equation (6) after y/R is calculated. An alternative is to use the Wallis equation.

A computer program was prepared in accordance with this solution procedure and the series 7 equations. Table 1 shows a comparison of calculated and experimental data for pressure drop and holdup as determined by several different investigators.

Holdup data are calculated for the liquid in the film plus entrained liquid with the assumption that homogeneous flow exists in the core. Existing holdup correlations in the range of $R_L < 0.10$ are poor as shown by Dukler, Wicks, and Cleveland (1964). This method appears to provide a satisfactory prediction as well as for pressure drop.

This data analysis represents a maximum tube diameter of about 4 cm and a maximum liquid viscosity of about 5 cp. Liquid data with high viscosity liquids indicate that Figure 2 applies to these flow conditions but that Figures 3 and 4 are not applicable.

NOTATION

a = core radius
D_c = diameter of gas core
f = friction factor
dP/dL = pressure gradient
R = pipe radius
R_L = liquid holdup fraction
Re_L = film Reynolds number $4\Gamma/\mu_L$
u_G = gas velocity in core
$u_G{}^*$ = shear velocity $[(\tau_w + \tau_i)/2\rho_G]^{1/2}$
$u_L{}^*$ = shear velocity $[(\tau_w + \tau_i)/2\rho_L]^{1/2}$
u_{SG} = gas superficial velocity
W_E, W_G = mass flow rate of entrainment and gas respectively
y = film thickness
$y_G{}^+$ = $y u_G{}^*/\nu_G$
$y_L{}^+$ = $y u_L{}^*/\nu_L$

Greek Letters

Γ = mass flow rate per unit wetted perimeter
μ_G, μ_L = viscosity of gas and liquid respectively
ν_G, ν_L = kinematic viscosity of gas and liquid respectively
ρ_G, ρ_L = density of gas and liquid respectively
τ_i = interfacial shear
τ_w = wall shear

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Gas-Liquid Interfacial Area Determination for a Turbine Agitated Reactor

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Gas absorption accompanied by a very rapid chemical reaction has been used as a method for the determination of interfacial area between gas and liquid phases. Methods used are oxidation of aqueous sodium sulfite solutions in the presence of a catalyst and carbon dioxide absorption in solutions of alkalis and amines. Visual observation of gas dispersions in mechanically agitated liquid systems indicates that the interfacial area in typical organic liquids is much greater than in aqueous systems. Thus there is an incentive to develop a chemical method for interfacial area determination for organic liquids. Selection of the system must consider the following requirements:

1. The reaction of gas and liquid must be very fast so as to provide a boundary layer reaction. Van Krevelin and Hoftyzer (1953) and Yoshida and Miura (1963) have shown that the theory of gas absorption with fast first-order chemical reaction in the liquid phase results in the apparent mass transfer coefficient

$$k_L^* = \sqrt{kD} \tag{1}$$

and $k_L^*/k_L > 2$. Calderbank (1959) reports typical values of k_L for agitated vessels are in the range of 0.01 to 0.1 cm/s and liquid phase diffusivity for gases is of the order of 3×10^{-5} cm^2/s so the kinetic rate must be greater than 10^3 s^{-1} to provide a boundary layer reaction.
2. The reaction between gas and the reacting liquid phase component must be capable of being represented by one first- or second-order reaction.
3. No gaseous reaction products are generated which will dilute the reactant gas phase and provide mass transfer resistance in the gas phase.
4. The reacting liquid phase component must have negligible vapor pressure at reaction temperature so that the reaction can be confined to the liquid phase.

This is a difficult list of qualifications to meet. Aluminum alkyls provide very fast reactions with oxygen to form aluminum alkoxides so C_{10} aluminum alkyl was selected. This material meets all of the requirements for interfacial area determination and can be mixed with nonreacting liquids to provide a range of liquid viscosity. The oxidation does have a disadvantage in the high heat of reaction, approximately 180 k cal/g mole of oxygen. The reaction proceeds too fast to follow with liquid phase analysis so a system was designed to determine the rate of oxygen consumption.

EXPERIMENTAL APPARATUS AND PROCEDURE

A 4-in. diam. pyrex vessel was used for all of these experiments. Four stainless steel baffles, each with a width of one-tenth of the tank diameter, were used. The impeller was 2-in. diam. with six flat blades. Liquid depth was equal to the vessel diameter. The impeller was driven by a magnetic drive which was controlled by a Cole-Parmer standard Servodyne. Oxygen was supplied from a 1.43-liter container fitted with a PP type SR-4 pressure cell and a pressure gauge. A recorder was used for the output signal from the pressure cell. The pressure cell has a maximum error of ¼% over the calibrated pressure range. The unit was calibrated before each run over a range of about two atmospheres so the maximum error in pressure was about 4 mm Hg.

Experimental procedures were as follows: The reactor system was leak tested at full vacuum from a vacuum pump. The system was pressurized with nitrogen and then the solvent was charged. The alkyl was then added under the liquid surface so that it would go to the bottom of the reactor. Initial alkyl concentration was 30% by volume. The nitrogen was evacuated and an absolute pressure of about 5 mm Hg was maintained while the liquid was degassed. This was accomplished by freezing and thawing the liquid phase under vacuum. Two freezing cycles appeared to remove the dissolved gas. The reactor system was purged with 99.5% oxygen under vacuum and was then pressurized to about 100 mm Hg pressure with 99.999% oxygen from the pressure cell bomb without agitation. A mercury manometer was used for pressure readings on the reactor. The impeller was then set at the desired speed and oxygen was fed from the pressure cell bomb to maintain a constant pressure. Pressure in the bomb as a function of time was recorded by the recorder. After about 10% oxidation of the alkyl, the oxygen was shut off and agitation was continued until a constant pressure reading was observed in the reactor. Residual inerts pressure was observed to be 10 to 20 mm Hg and this was found to